Thermal Casimir effect with soft boundary conditions

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We consider the thermal Casimir effect in systems of parallel plates coupled to a massless free field theory via quadratic interaction terms which suppress (i) the field on the plates and (ii) the gradient of the field in the plane of the plates. These boundary interactions correspond to (i) the presence of an electrolyte in the plates and (ii) a uniform field of dipoles, in the plates, which are polarizable in the plane of the plates. These boundary interactions lead to Robin-type boundary conditions in the case where there is no field outside the two plates. In the appropriate limit, in both cases Dirichlet boundary conditions are obtained but we show that in case (i) the Dirichlet limit breaks down at short interplate distances and in (ii) it breaks down at large distances. The behavior of the two plate system is also seen to be highly dependent on whether the system is open or closed. In addition we analyze the Casimir force on a third plate placed between two outer plates. The force acting on the central plate is shown to be highly sensitive to whether or not the fluctuating scalar field is present in the region exterior to the two confining plates.

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I. INTRODUCTION

The Casimir effect is often described in terms of how a boundary condition modifies the fluctuations of a field [1,2], the classic example being the case of the modification of the vacuum energy of the electromagnetic field between two conducting plates. However, boundary conditions often arise from the consideration of ideal media such perfect conductors. In reality the Casimir force is generated by interactions of the plates via the electromagnetic field, the material properties of the plates being coupled to the field. This point of view is embodied in the Lifshitz formulation of van der Waals interactions between macroscopic bodies [3]. Also in the study of the critical Casimir force energetic boundary terms arise naturally in spin models due to surface interactions and fields [4,5]. In this paper we analyze the fluctuation induced interactions due to a free massless field theory in the presence of planes with quadratic interactions in the field variable. In an electrostatic analogy one case is equivalent to the field interacting with dipoles confined to the plane and the other case is equivalent to an electrolyte, in the Debye Hückel limit, confined to the plates. In the limits where the dipole polarizability becomes infinite or the concentration of electrolyte becomes infinite, the limiting boundary conditions are Dirichlet. Clearly these two limiting cases mimic a conducting plate limit but via two distinct physical mechanisms. Here we show that the limit in which the Dirichlet limit is valid, for large but finite dipole or electrolyte strengths, depends on the model. In the electrolyte case deviations from the Dirichlet limit become apparent at short interplane separations but in the dipole case deviations appear for large interplane separations. We also compare the results for two planes where the field exists in the region outside—the open system—with the case where the field does not exist outside—the closed system. This latter case corresponds to that arising in studies of the critical Casimir effect where the order parameter field exist within the critical fluid but not outside the boundaries of the physical system. The Casimir force in this case can be attractive or repulsive depending on the boundary conditions at the two confining plates. However, we show that when the fluctuating medium exists outside the two plates then the interaction is always attractive. The case of a third plate confined between two other plates is also studied in both the open and closed systems. Here the force acting on the third plate can be evaluated and we find a rich behavior and striking qualitative differences between the force on the central plate in the closed and open system. The method we use to carry out the computations is based on a path integral method adapted to planar geometries introduced in Ref. [6]. The computations in this formalism are very short and straightforward and also have the advantage of highlighting immediately the differences between open and closed systems.

II. MODEL AND TWO PLATE INTERACTION

We consider a free scalar field theory analogous to that occurring for electrostatics with a free kinetic term everywhere in space but with additional interaction terms with two surfaces at $z=0$ and $z=l$

$$H=\frac{1}{2}\int_V d\mathbf{x}[\nabla\phi(\mathbf{x})]^2+\int_V d\mathbf{x}\,\delta(z)f_1[\phi(\mathbf{x})]+\int_V d\mathbf{x}\,\delta(z-l)f_2[\phi(\mathbf{x})]. \quad (1)$$

The terms f_1 and f_2 are functionals of the field ϕ on the two surfaces. Here we distinguish between the coordinate z perpendicular to the plates and the coordinates perpendicular to the z direction denoted by $\mathbf{x}_\perp$. In this notation therefore any point is given by coordinate $\mathbf{x}=(z,\mathbf{x}_\perp)$. The field ϕ is defined on a region of space with $\mathbf{x}_\perp$ in a plane of area A and z in the region $[-L,L]$ and we will be interested in the thermodynamic limits as $A\to\infty$ and $L\to\infty$. Note that the open system corresponds to the so-called defect plane case [4], as opposed to the usual case considered in boundary critical phenomena where the field only exists in the region $[0,l]$—physically the two cases are quite different as we shall see when comparing our results to some results in the literature [4,7–10].

We will consider two types of interaction terms. First the case where the field ϕ is acquires a mass in the plates (type I), i.e., it has a harmonic self-interaction

$$f_i[\phi(\mathbf{x})] = \frac{c_i}{2}\phi^2(\mathbf{x}). \tag{2}$$

This sort of interaction, for c_i positive, arises naturally in the Debye Hückel theory of electrolytes and the coefficient c_i is proportional to the electrolyte concentration (for example, see Ref. [6]). When the c_i are positive this term will suppress the amplitude of the field ϕ at the plates and we expect that in the limit $c_i \to \infty$ we will recover Dirichlet boundary conditions. We could also consider the case where the gradient of the field ϕ in the plane is energetically suppressed (type II) via

$$f_i[\phi(\mathbf{x})] = \frac{\chi_i}{2}[\nabla_\perp \phi(\mathbf{x})]^2, \tag{3}$$

in this case $\mathbf{E}_\perp = -i\nabla_\perp \phi$ is suppressed and is set to zero in the limit $\chi_i \to \infty$. This is the boundary condition for an electric field on a conductor. Clearly in both cases (up to an irrelevant zero mode) the boundary conditions for the two cases become equivalent in the limit $c_i, \chi_i \to \infty$. The purpose of this paper is to explore the modifications of the Casimir the effect when the coefficients c_i are finite. The boundary interaction term in Eq. (3) actually occurs quite naturally in a model of surfaces containing dipoles whose dipole moments are constrained to lie within the plane of the plates. The electrostatic Hamiltonian is now given by

$$\begin{aligned} H = {} & \frac{1}{2}\int_V d\mathbf{x}[\nabla\phi(\mathbf{x})]^2 + i\int_V d\mathbf{x}\,\delta(z)\nabla_\perp\phi(\mathbf{x})\cdot\mathbf{P}_1(\mathbf{x}_\perp) \\ & + i\int_V d\mathbf{x}\,\delta(z-l)\nabla_\perp\phi(\mathbf{x})\cdot\mathbf{P}_2(\mathbf{x}_\perp) + \frac{1}{2\chi_1}\int_A d\mathbf{x}_\perp\cdot\mathbf{P}_1^2(\mathbf{x}_\perp) \\ & + \frac{1}{2\chi_2}\int_A d\mathbf{x}_\perp\cdot\mathbf{P}_2^2(\mathbf{x}_\perp), \end{aligned} \tag{4}$$

where $-i\nabla\phi$ is the electric field and $\mathbf{P}_{1,2}$ represent uniform dipole fields, constrained to lie within the plane of the plates, of polarizabilities $\chi_{1,2}$. Now integrating the corresponding partition function over the fields $\mathbf{P}_1$ and $\mathbf{P}_2$, yields an effective Hamiltonian for the field ϕ with surface interaction terms of the form (3).

We note that the classical equation of motion for the field in the electrolyte case induces Robin-type boundary conditions at the surface relating the jump of the field derivative in the z direction to the value of the field on the surface. In the case where there is no space in the region external to the two plates, standard one-sided Robin boundary conditions are obtained and this case has been extensively studied in the literature [7–10].

In this paper we will use a calculational technique, based on the Feynman path integral method which has been adapted to study a variety of problems in electrostatics systems, the thermal Casimir effect and membrane fluctuations [6,11,12]. We proceed by decomposing the field ϕ into its Fourier components in the plane of $\mathbf{x}_\perp$, i.e., we write

$$\phi = \frac{1}{\sqrt{A}}\sum_{\mathbf{k}} \tilde{\phi}(\mathbf{k},z)\exp(i\mathbf{k}\cdot\mathbf{x}_\perp). \tag{5}$$

The Hamiltonian is now given by

$$\begin{aligned} H = \sum_{\mathbf{k}} \Bigg\{ & \frac{1}{2}\int dz\left[\frac{d\tilde{\phi}(\mathbf{k},z)}{dz}\frac{d\tilde{\phi}(-\mathbf{k},z)}{dz} + \mathbf{k}^2\tilde{\phi}(\mathbf{k},z)\tilde{\phi}(-\mathbf{k},z)\right] \\ & + \frac{1}{2}g_1(\mathbf{k})\tilde{\phi}(\mathbf{k},0)\tilde{\phi}(-\mathbf{k},0) + \frac{1}{2}g_1(\mathbf{k})\tilde{\phi}(\mathbf{k},l)\tilde{\phi}(-\mathbf{k},l)\Bigg\}. \end{aligned} \tag{6}$$

In the case of the scalar interaction (type I) term of Eq. (2) we have that

$$g_i(\mathbf{k}) = c_i, \tag{7}$$

and in the case of the transverse field (type II) interaction term of Eq. (3) we have

$$g_i(\mathbf{k}) = \chi_i k^2, \tag{8}$$

where $k = |\mathbf{k}|$. The resulting field theory is noninteracting and the modes are all decoupled; we may thus write the partition function as a product over the partition function of the modes

$$\ln(Z) = \sum_{\mathbf{k}} \ln(Z_{\mathbf{k}}), \tag{9}$$

with

$$\begin{aligned} Z_{\mathbf{k}} = \int d[X_k]\exp\Bigg[& -\frac{\beta}{2}\int dz\left(\frac{dX_k^2}{dz} + k^2X_k^2\right) \\ & -\frac{\beta}{2}g_1(k)X_k(0)^2 - \frac{\beta}{2}g_2(k)X_k(l)^2\Bigg], \end{aligned} \tag{10}$$

where we have decomposed the field $\tilde{\phi}$ into its real and imaginary parts and as usual we only take half the sum over the modes $\mathbf{k}$ as the field ϕ is real. Each partition function has the form of a simple harmonic oscillator path integral with interaction terms inserted at the *times* $z=0$ and $z=l$. The path integral kernel defined as

$$\begin{aligned} K(x,y,z_1,z_2,\omega,M) = {} & \int_{X(z_1)=x}^{X(z_2)=y} d[X] \\ & \times\exp\left[-\frac{M}{2}\int_{z_1}^{z_2} dz\left(\frac{dX^2}{dz} + \omega^2X^2\right)\right] \end{aligned} \tag{11}$$

is given explicitly as

$$\begin{aligned} K(x,y,z_1,z_2,\omega,M) = {} & \left(\frac{M\omega}{2\pi\sinh[\omega(z_2-z_1)]}\right)^{1/2} \\ & \times\exp\Bigg(-\frac{1}{2}(x^2+y^2)M\omega\coth[\omega(z_1-z_2)] \\ & + xyM\omega\,\mathrm{cosech}[\omega(z_1-z_2)]\Bigg). \end{aligned} \tag{12}$$

We now note that in the limit $(z_2 - z_1) \to \infty$

$$K(x,y,z_1,z_2,\omega,M) \approx \left(\frac{M\omega}{\pi}\right)^{1/2} \exp\left(-\frac{1}{2}\omega(z_2-z_1)\right) \times \exp\left(-\frac{M\omega}{2}(x^2+y^2)\right). \quad (13)$$

Thus the initial and final positions become decoupled. Therefore in the limit $L\to\infty$, up to arbitrary terms depending on the values of the field $x(-L)$ and $x(L)$ we find

$$Z_k = \left(\frac{\beta k}{\pi}\right)\exp\left(-kL+\frac{1}{2}kl\right)\int dxdy\,\exp\left(-\frac{\beta}{2}x^2 \times[k+g_1(k)]\right)K(x,y,0,l,k,\beta)\exp\left(-\frac{\beta}{2}y^2[k+g_2(k)]\right). \quad (14)$$

This is a trivial Gaussian integral to do and we find that, up to bulk terms denoted here by B_k (independent of l and the g_i), we have

$$\ln(Z_k) = B_k - \frac{1}{2}\Bigg[\ln[2k+g_1(k)] + \ln[2k+g_2(k)] + \ln\left(1-\frac{g_1(k)g_2(k)\exp(-2kl)}{[2k+g_1(k)][2k+g_2(k)]}\right)\Bigg]. \quad (15)$$

The first term, as mentioned above, is a bulk term, the first term in the square bracket is a surface energy term for each surface and the final term is the l-dependent term giving rise to the Casimir interaction. The l-dependent Casimir free energy is thus given, in a space of total dimension d, by

$$\frac{F_o(l)}{A} = \frac{k_BT}{2}\int\frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}}\ln\left(1-\frac{g_1(k)g_2(k)\exp(-2kl)}{[2k+g_1(k)][2k+g_2(k)]}\right) = \frac{k_BT}{(4\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right)} \times \int k^{d-2}dk\,\ln\left(1-\frac{g_1(k)g_2(k)\exp(-2kl)}{[2k+g_1(k)][2k+g_2(k)]}\right), \quad (16)$$

where Γ is the Euler gamma function and the subscript o is to remind us that this is result for an open system.

The first thing to notice is that in the strict limits $g_1\to\infty$ and $g_2\to\infty$ we recover the classical result for Dirichlet boundary conditions

$$\frac{F_D(l)}{A} = -\frac{k_BT\Gamma(d-1)\zeta(d)}{(16\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right)l^{d-1}}, \quad (17)$$

where ζ is the Riemann zeta function. In the limit where one of the g_i is zero then the result is zero as it should be—this is a critical difference between the case of defect planes where the field exists outside the interior of the plates and the case where it does not exist outside the plates. We note that in a finite or closed system, where the field ϕ does not exist outside the two plates as is the case in studies of the critical Casimir force [7–10], the result is somewhat different. Indeed in this case it is possible to have repulsive as well as attractive regimes, and moreover there is a residual interaction even in the case where one of the c_i is set to zero. In this open case the interaction is always attractive and it vanishes when either of the c_i, not just both, is set to zero. The results of Refs. [7–10] can easily be recovered in our formalism. When there is no region exterior to the slab the external propagators are absent and thus the ground state wave function at each interface is not there. This means that g_i (which is added to k in our case) is simply replaced by g_i-k. Upon subtraction of the bulk pressure one thus finds a Casimir force for a closed (hence a subscript c) system given by

$$\frac{F_c(l)}{A} = \frac{k_BT}{(4\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right)} \times \int k^{d-2}dk\,\ln\left(1-\frac{[g_1(k)-k][g_2(k)-k]\exp(-2kl)}{[k+g_1(k)][k+g_2(k)]}\right), \quad (18)$$

in agreement with the results of Refs. [7–10]. Note that it is the appearance of the terms g_i-k in the above expression that give the possibility of repulsive Casimir interactions [7–10]. The appearance of a repulsive interaction is most easily seen in the limit $g_1\to\infty$ and $g_2\to 0$. However we reemphasize that the presence of the field in the exterior region ensures that the interaction is always attractive (for g_i positive).

We now return to the case where the g_i are finite, for the surface interaction term of Eq. (3) (type II) we find that

$$\frac{F_o(l)}{A} = \frac{k_BT}{(4\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right)} \times \int k^{d-2}dk\,\ln\left(1-\frac{\chi_1\chi_2k^2\exp(-2kl)}{(2+\chi_1k)(2+\chi_2k)}\right). \quad (19)$$

Clearly in the large l limit the integral above is dominated by the small k behavior and thus for sufficiently large l the asymptotic behavior of the free energy is given by

$$\frac{F_o(l)}{A} = \frac{k_BT}{(4\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right)} \times \int k^{d-2}dk\,\ln\left(1-\frac{\chi_1\chi_2k^2\exp(-2kl)}{4}\right). \quad (20)$$

Thus at sufficiently large l the Dirichlet limit is no longer valid and the Casimir free energy will become dependent on the χ_i. In this limit we thus find

$$\frac{F_o(l)}{A} = -\frac{k_BT\Gamma(d+1)\chi_1\chi_2}{16(16\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right)l^{d+1}}, \quad (21)$$

and thus the strength of the interaction is considerably reduced. The large k part of the integral dominates the short distance behavior and the interaction thus remains of the Dirichlet form in this regime. The cross over length between Dirichlet and this modified long distance behavior is given by $l_c \sim \chi$ if the two χ_i are of the same order. If χ_1 and χ_2 are very different then there is an even richer behavior and it is possible to have an intermediate regime where $F/A \sim -1/l^d$.

Now we consider the case of the (type I) surface interaction (2), here we find that the small k limit agrees with the Dirichlet limit and thus the long distance behavior of the interaction in this case is of the Dirichlet form. The fact that the Dirichlet limit for type-I interactions holds at large l is a consequence of the fact that $c_i=\infty$ is a infrared stable fixed point (for both the free and interacting field theories) [4]. However, the deviations from the Dirichlet case are seen for large k and thus will show up in the short distance behavior of the interaction. In this case the Casimir pressure is given by given by

$$P_o(l) = -\frac{\partial}{\partial l}\frac{F_o(l)}{A} = -\frac{k_B T c_1 c_2}{2(16\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right) l^{d-1}} \times \int u^d du \frac{\exp(-u)}{(u+c_1 l)(u+c_2 l) - c_1 c_2 l^2 \exp(-u)}. \tag{22}$$

In the limit of large l the Dirichlet limit is clearly always good, however, it breaks down at small l when $l \ll 1/c_i$. In this limit of small l we obtain (for $d \geqslant 2$)

$$P_o(l) = -\frac{k_B T c_1 c_2 \Gamma(d-1)}{2(16\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right) l^{d-1}}. \tag{23}$$

It is easy to verify that this is a reduction of the Casimir pressure with respect to the ideal Dirichlet case in the limiting region where it is valid.

III. THREE PLATE INTERACTION

In order to further demonstrate the power of the path integral method in the context of Casimir interaction we will consider the case of three plates. We keep two plates [plates (1) and (3)] at $z=0$ and $z=l$ and we will place another plate between them at $z=m$. Again we denote the quadratic surface interaction coefficients by $g_i(k)$ where i is the plate number. The computation for this case within the path integral formalism is immediate (it encodes to a certain extent the transfer matrix formalism developed for van der Waals interactions in slab geometries developed in Ref. [13]). The partition function for the mode $Z_{\mathbf{k}}$ is given by

$$Z_{\mathbf{k}} = \left(\frac{\beta k}{\pi}\right)\exp\left(-kL + \frac{1}{2}kl\right) \times \int dxdydz \exp\left(-\frac{\beta}{2}x^2[k+g_1(k)]\right) \times K(x,y,0,m,k,\beta)\exp\left(-\frac{\beta}{2}y^2 g_2(k)\right) \times K(y,z,m,l,k,\beta)\exp\left(-\frac{\beta}{2}z^2[k+g_3(k)]\right). \tag{24}$$

This yields

$$\ln(Z_k) = B_k - \frac{1}{2}\{\ln[2k+g_1(k)] + \ln[2k+g_2(k)] + \ln[2k+g_3(k)]\} - \frac{1}{2}\ln\left[1 - \frac{g_1(k)g_2(k)\exp(-2km)}{[2k+g_1(k)][2k+g_2(k)]} - \frac{g_2(k)g_3(k)\exp(-2km')}{[2k+g_2(k)][2k+g_3(k)]} + \frac{g_1(k)g_3(k)(g_2(k)-2k)\exp[-2k(m+m')]}{[2k+g_1(k)][2k+g_2(k)][2k+g_3(k)]}\right], \tag{25}$$

where $m'=l-m$. The first term is a bulk energy independent of m and m', the second corresponds to three independent surface energies and the third contains the geometry dependent interaction. An important test of the above is that upon setting $g_2=0$ we recover the two plate result of Eq. (15). The l (geometry) dependent part of the Casmir free energy is given by

$$\frac{F_o(m,m')}{A} = \frac{k_B T}{(4\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right)} \times \int k^{d-2} dk \ln\left[1 - \frac{g_1(k)g_2(k)\exp(-2km)}{[2k+g_1(k)][2k+g_2(k)]} - \frac{g_2(k)g_3(k)\exp(-2km')}{[2k+g_2(k)][2k+g_3(k)]} + \frac{g_1(k)g_3(k)[g_2(k)-2k]\exp[-2k(m+m')]}{[2k+g_1(k)][2k+g_2(k)][2k+g_3(k)]}\right]. \tag{26}$$

If we take the Dirichlet limit $g_i \to 0$ for all i we obtain that the free energy is given by

$$F_D(m,m') = F_D(m) + F_D(m'), \tag{27}$$

i.e., the sum of the free energies of two independent systems with Dirichlet boundary conditions whose values are given by Eq. (17). This result is clearly expected on physical grounds as strict Dirichlet boundary conditions effectively decouple to two systems (plate 1 and 2 and plate 2 and 3). However in the general case we see that there is no decoupling and that n-body (plate) interactions are important. We also notice that the free energy also becomes equal to the sum of two independent terms (one dependent on m and the other on m') in the limit where $g_2 \to \infty$.

The case where the system is closed (no exterior field) can also be analyzed as before. Here we find (simply by replacing $g_{1,3}$ by $g_{1,3}-k$ and leaving g_2 unchanged)

$$\frac{F_c(m,m')}{A} = \frac{k_B T}{(4\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right)} \int k^{d-2} dk \ln\Bigg[1 - \frac{[g_1(k)-k]g_2(k)\exp(-2km)}{[k+g_1(k)][2k+g_2(k)]} - \frac{g_2(k)[g_3(k)-k]\exp(-2km')}{[2k+g_2(k)][k+g_3(k)]} + \frac{[g_1(k)-k][g_3(k)-k][g_2(k)-2k]\exp[-2k(m+m')]}{[k+g_1(k)][2k+g_2(k)][k+g_3(k)]}\Bigg]. \quad (28)$$

We see that as long as g_1 and g_3 are finite then the results for the open and closed systems are quite different.

Let us consider the case of type-I boundary terms. For the case where the two outermost plates are fixed at a distance 1 let us define by

$$V_{c,o}(x) = F_{c,o}(x,1-x) = A\frac{k_B T}{(4\pi)^{(d-1)/2}\Gamma\left(\frac{d-1}{2}\right)} v_{c,o}(x), \quad (29)$$

the effective potential felt by the central plate (plate 2). We restrict ourselves to the symmetric case $c_1=c_3$ and which we will vary and we take $c_2=1$, also we shall consider the case $d=3$. Shown in Figs. 1(a) and 1(b) are the scaled effective potentials v (evaluated by numerical integration) for the cases of open (solid line) and closed systems for $c_1=2$ (a) $c_1=c_3=10$ (close to the Dirichlet limit for the external plates). We see that for $c_1=c_3=2$ and $c_2=10$ that for an open system the middle plate is always attracted to the exterior plates. However for a closed system for $c=2$ the middle plate is repelled from the two exterior plates and actually has an equilibrium position at the center of the two plates. For $c=10$ the closed system has a potential which is close to that of the open system near the middle of the two plates and the midpoint is an unstable equilibrium point in both open and closes systems. However, the closed system develops a repulsive potential close to the plates leading to a stable potential minima close to each plate. Notice that in the case of $c_1=10$ that the deviations from Dirichlet behavior for the closed system are manifested when the distance between the central plate and the closest bounding plate is small, this should be expected from our discussion in Sec. II. If we consider the case of type-II boundary terms we expect that deviations from the Dirichlet limit occur at large distances, We therefore consider a system of three plates again with a distance of 1 between the bounding plates. This distance should be large to see an effect and this is achieved by setting the polarizabilites χ to be small. Shown in Figs. 2(a) and 2(b) are the scaled effective potentials v for the open (solid lines) and closed (dashed lines) systems. In Fig. 2(a) we have set $\chi_1=\chi_3=0.1$ and $\chi_2=0.1$. In the case of an open system the central position is unstable and the middle plane is attracted towards the outer plates. However, for a closed system the central point is metastable and there is an energy

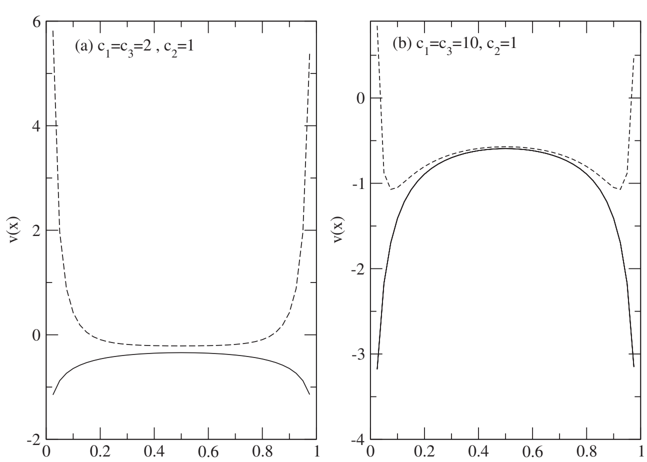

FIG. 1. Effective potential felt by a plane in the middle of two fixed planes all with type-I boundary interactions. Solid lines for open systems and dashed lines for closed systems.

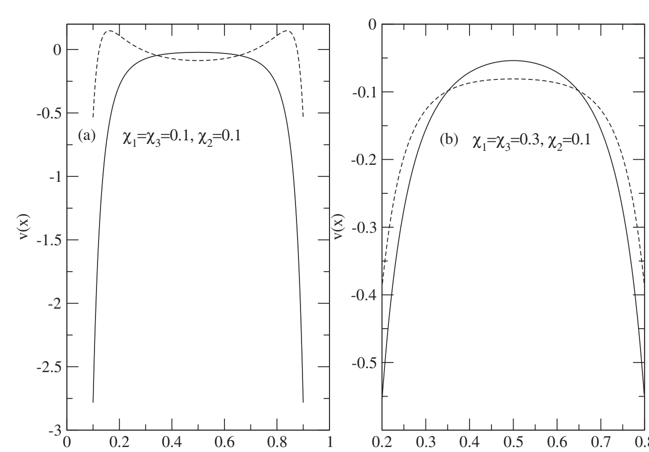

FIG. 2. Effective potential felt by a plane in the middle of two fixed planes all with type-II boundary interactions. Solid lines for open systems and dashed lines for closed systems.

barrier which must be crossed to reach the walls (which are ultimately attractive). If $\chi_1=(\chi_3)$ is increased the local minima at the midpoint eventually disappears, as shown in Fig. 2(b), where we have taken $\chi_1=0.3$, and the two curves for the open and closed systems are qualitatively the same.

IV. CONCLUSION

In this paper we have studied a free field scalar theory in the presence of planes which interact quadratically with the field. In one case (type I) the field acquires a mass on the plane which suppresses its fluctuations. This would correspond to the way in which an electrolyte confined in the plane interacts with the thermal fluctuations of the electrostatic field. The second term (type II) is proportional to the square of the in plane gradient and arises due to dipole interactions with the electrostatic field. In both of these cases if the strength of the interaction is taken strictly to infinity we obtain Dirichlet boundary conditions. We have seen, however, that for finite interactions the interaction between the two plates deviates from the Dirichlet behavior, at small distances for type I and at large distances for type II and no longer has a universal form. We have also seen that for finite interaction terms there is a clear difference between open systems (where the fluctuating field exists outside the two plates) and closed systems (where there is no fluctuating field outside the plates). Notably for open systems the interactions between plates are always attractive, this is in contrast to the case of closed systems where it has been long established that both attractive and repulsive interactions are possible [7–10]. We have also examined the behavior of a third plane sandwiched between two other planes, this demonstrates clearly the power of the path integral method used to analyze Casimir-type interactions in planar systems. Again, whether or not the system is open or closed can have a drastic influence on the force experienced by the third (central) plane. In closed systems the force felt by the central plane can be attractive or repulsive and even change sign in the same system, having positions of local equilibria away from the bounding walls (both stable and metastable). There are clearly many other configurations and setups that one can study with the formalism developed here and it is possible that some of the basic mechanisms seen here can be exploited in the design of nanodevices [14] where Casimir-type forces such as van der Waals interactions play an important role.

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